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Effects of Planetary Gear Ratio on Mean Service Life

M. Savage
University of Akron
Akron, Ohio

K.L. Rubadeux
The Hendrickson Company
Canton, Ohio

and

H.H.Coe
Vehicle Propulsion Directorate
U.S. Army Research Laboratory
Lewis Research Center
Cleveland, Ohio

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M. Savage
The University of Akron
Akron, Ohio 44325

K. L. Rubadeux
The Hendrickson Company
Canton, Ohio 44707

H. H. Coe
NASA Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

Planetary gear transmissions are compact, high-power speed reductions which use parallel load paths. The range of possible reduction ratios is bounded from below and above by limits on the relative size of the planet gears. For a single plane transmission, the planet gear has no size at a ratio of two. As the ratio increases, so does the size of the planets relative to the sizes of the sun and ring. Which ratio is best for a planetary reduction can be resolved by studying a series of optimal designs. In this series, each design is obtained by maximizing the service life for a planetary with a fixed size, gear ratio, input speed, power and materials. The planetary gear reduction service life is modeled as a function of the two-parameter Weibull distributed service lives of the bearings and gears in the reduction. Planet bearing life strongly influences the optimal reduction lives which point to an optimal planetary reduction ratio in the neighborhood of four to five.

NOMENCLATURE

Variables

- a - gear addendum (mm,in) and bearing life adjustment factor
- C - dynamic capacity (kN,lbs)
- E - elastic modulus (MPa,psi)
- f - face width (mm,in)
- F - load (kN,lbs)
- K_f - stress concentration factor
- ℓ - life (106 cycles and hours)
- n - gear ratio relative to the arm, number of planets
- n_a - actual transmission gear ratio
- N - number of gear teeth
- P_d - diametral pitch (1.0/inch)
- R - gear radius (mm,in) and reliability
- Y - Lewis Form Factor
- ϕ - pressure angle (degrees, radians)
- ν - Poisson's Ratio
- ρ - radius of curvature (mm,in)
- σ - bending stress (Pa, psi)
- σ_H - Hertzian contact stress (Pa, psi)
- ω - angular velocity, speed (rpm)
- ω_b - bearing load cycle speed (rpm)
- Σ - central angle between two adjacent planet center lines with the input shaft center (radians)

Subscripts

- av - mean
- d - dynamic
- o - output
- pl - planet
- r - ring gear
- s - sun gear
- 1 - pinion
- 2 - gear
- 10 - 90 percent reliability

Superscripts

- b - Weibull slope exponent
- p - load-life exponent

INTRODUCTION

Planetary gear transmissions offer the user a moderate gear reduction with a high power density (Fox,1991) (Lynwander,1983). By carrying multiple planet gears on a rotating arm, load sharing is enabled among the planets (Fox,1991) (Lynwander,1983). The symmetric placement of the planets about the input sun gear provides radial load cancellation on the bearings which support the input sun and the output arm (Fox,1991) (Lynwander,1983). The fixed internal ring gear support also has no net radial load. With near-equal load sharing in medium to fine pitch gearing, a compact reduction results. Planetary reductions are often found in transportation power transmissions due to this weight and volumetric efficiency (Fox,1991) (Rashidi & Krantz,1992).

Much of the published design literature for planetary gearing focuses on the kinematic proportioning of the unit to achieve one or more reductions through the auspicious use of clutches and brakes (Muller,1982) (Tsai,1987).

Recent literature on planetary gears has focused on the dynamic loads in the transmission with measurements of load sharing and the variations of load in specific units (Rashidi & Krantz,1992) (Donley & Steyer,1992) (Saada & Velex,1995) (Kahraman,1994). Monitoring of the dynamic loads in a planetary has also been proposed as one method of determining the need for preventive maintenance in the transmission (Chin, et al,1995).

While the reduction of dynamic loads in a planetary transmission is an important task, these studies do not indicate which ratio is best suited for a planetary transmission. Studies of rotating power in the planetary have indicated that as the ratio is increased, the percent of rotating power in the unit reduces (Lynwander,1983). This suggests that the best ratio for a planetary reduction is the highest possible, which is reached with the largest planet gears. Addendum interference between the planets determines this limit. However, when one considers the size of a planetary reduction required to transmit a given power level at some input speed, the loading on the gears and bearings in the reduction become an important factor as do the component lives under load (Savage, et al,1989 & 1994a).

Since aircraft and automotive transmissions can see service in excess of their nominal design lives, periodic maintenance is provided throughout their lives (Chin, et al,1995) (Savage & Lewicki,1991). The this work are single plane reductions with input sun gears, fixed ring gears and multiple planet gears. The planet gears are placed symmetrically about the concentric input and output shafts as shown in Figure 1. Each planet of a reduction is connected to the output arm through a single, ball bearing at its center. Since the input sun gear and fixed ring gear mesh with all the planet gears, a single diametral pitch or module is used for all gears in a reduction as is a single face width.

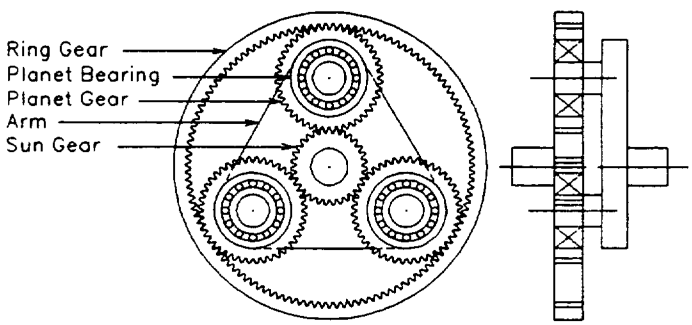


Figure 1.
Single Plane Planetary Transmission

No bearings are included on the input or output shaft since the internal loads in the planetary are balanced on these shafts due to the symmetric placement of the planets. Bearings are needed on these shafts but their placement and loading are based on external considerations.

All transmissions carry the same power and have the same outside diameter which provides a radial ring thickness outside the ring gear teeth of 1.5 times the tooth height.

In this comparison, the input speed and torque are fixed as the ratio is varied. For each design, the planetary system life is maximized subject to the above constraints in addition to constraints on the stresses in the gear teeth and on assembly clearances. The parameters which define each design are the number of teeth on the sun gear, N_s, the face width of the gears, f, and the diametral pitch of the gears, P_d.

where the overall transmission gear ratio relative to the arm is:

$$n = n_1 \cdot n_2 \tag{3}$$

and the speed of the output arm relative to the fixed ring is:

$$\omega_o = \frac{\omega_s}{(1-n)} \tag{4}$$

So the planetary gear ratio is:

$$n_a = 1 - n \tag{5}$$

And the speed of each planet gear is:

$$\omega_{pl} = \omega_s \left(\frac{n_1 - n}{n_1(1-n)}\right) = \omega_s \left(\frac{1-n_2}{1-n}\right) \tag{6}$$

The planet bearing load cycle speed is the speed of the planet with respect to the arm:

$$\omega_b = \omega_{pl} \left(\frac{n}{n - n_1}\right) \tag{7}$$

For each transmission studied, the planetary gear ratio, n_a, is fixed and the number of teeth on the sun gear is an independent design parameter. Values for the number of teeth on the sun gear, the gear face width and the diametral pitch are found which maximize the service life for a given transmission size. This requires the number of teeth on the ring gear, N_r, and on each planet gear, N_{pl}, to be found in terms of n_a and N_s.

The number of teeth on the ring gear is related to the number of teeth on the sun by the gear ratio relative to the arm, since the planets become idlers in this inversion.

$$N_r = -nN_s = (n_a - 1)\,N_s \tag{8}$$

Since the diameter of the ring gear is equal to the diameter of the sun gear plus twice the diameter of the planet gear, the number of teeth on each planet gear can be calculated by:

$$N_{pl} = \frac{N_r - N_s}{2} = \frac{(n_a - 1 - 1)\,N_s}{2} = \frac{(n_a - 2)\,N_s}{2} \tag{9}$$

To keep the number of planet teeth positive, the transmission gear ratio, outside diameter of the planet gear by twice the tooth addendum will accomplish this:

$$2 \cdot (R_s + R_{pl}) \cdot \sin\left(\frac{\Sigma}{2}\right) > 2 \cdot (R_{pl} + 2 \cdot a) \tag{10}$$

where Σ is the central angle between two adjacent planet center lines and a is the addendum of the planet gears.

One additional constraint is needed to allow the planets to be positioned symmetrically around the sun gear. The sum of the number of teeth on the sun and on the ring divided by the number of planets must produce an integer.

$$\frac{N_s + N_r}{n_{pl}} = I \tag{11}$$

TOOTH STRENGTH

The AGMA model for gear tooth bending uses the Lewis form factor and a stress concentration factor to determine the stress in the tooth for a load at the highest point of single tooth contact (AGMA,1988). The bending stress model is:

$$\sigma = \frac{F_d \cdot P_d \cdot K_f}{f \cdot Y} \tag{12}$$

where F_d is the tangential dynamic load on the tooth, K_f is the stress concentration factor, and Y is the Lewis form factor based on the geometry of the tooth. Since the Lewis form factor is a function of the tooth shape, it is dependent on the number of teeth on the gear, as is the stress concentration factor.

Large localized stresses occur in the fillets of gear teeth due to the change in the cross-section of the tooth. Although the maximum stress is located closer to the root circle than predicted by Lewis' parabola, the distance between the two locations of maximum stress is relatively small and the stress concentration factor accurately compares the maximum stress in the tooth to the Lewis stress (AGMA,1988). This method of rapid calculation of bending stress for external gear teeth is extended to include the bending stress in the internal gear teeth of the ring gear (Savage, et al,1995).

In addition to bending stresses, surface contact stresses can contribute to gear teeth failures. The Hertzian pressure model closely predicts these contact pressures:

$$\sigma_H = \left(\frac{F_d}{\pi \cdot f \cdot \cos\phi}\left(\frac{\frac{1}{\rho_1} + \frac{1}{\rho_2}}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}}\right)\right)^{1/2} \tag{13}$$

contact, v_1 and v_2 are the Poisson ratios and E_1 and E_2 are the moduli of elasticity of the materials for the two gears.

Contact pressure near the pitch point leads to gear tooth pitting which limits the life of the gear tooth. Gear tip scoring is another type of failure which is affected by the contact pressure at the gear tooth tip. One model for gear tip scoring includes the pressure times velocity factor, where the sliding velocity at the gear tip is tangent to the tooth surfaces.

SERVICE LIFE

Surface pitting due to fatigue is the basis for the life model for the bearings, gears and transmission. Fatigue due to this mode of failure has no endurance limit, but has a service life described by a straight line on the log stress versus log cycle S-N curve. This life to load relationship can be written for a specific load, F, at which the ninety-percent reliability life is ℓ_{10} and which is related to the component dynamic capacity, C, as:

$$\ell_{10} = a\left(\frac{C}{F}\right)^{p} \tag{14}$$

Here the component dynamic capacity, C, is defined as the load that produces a life of one-million cycles with a reliability of ninety-percent and a is the life adjustment factor. The power, p, is the load-life exponent which is determined experimentally.

Complementing this load-life relationship, is the two-parameter Weibull distribution for the scatter in life. In this distribution, the reliability, R, is related to the life, ℓ, as:

$$Ln\left(\frac{1}{R}\right) = Ln\left(\frac{1}{0.9}\right)\cdot\left(\frac{\ell}{\ell_{10}}\right)^{b} \tag{15}$$

A meaningful estimation of service time is the mean time between

$$\ell_{av,s} = \frac{1}{\sum \frac{1}{\ell_{av,i}}} \tag{17}$$

PLANETARY DESIGNS

In considering the effects of the gear ratio on the mean transmission life, the input speed and power were held constant. The input speed was 2,000 RPM for all transmissions which carried a power of 51 horsepower with a fixed input torque of 1,600 pound inches. Each transmission had a maximum ring gear outside diameter of 12 inches. The sun gear mesh and the ring gear mesh both had a normal pressure angle of 20 degrees and the same diametral pitch. All gears were made of high strength steel with a surface material strength of 220 ksi. The Hertzian contact pressure was limited to be less than 180 psi and the tooth bending stresses were limited to be less than 40 ksi. These limits include a total load design factor of 1.5 to adjust the nominal stress calculations of Eq. (12) and (13) to code levels. The PV factor was limited to be less than 50 million psi-ft/min and the gear tooth flash temperature was limited to be less than 200° F. The Weibull slope of the sun gear, the three planet gears, and the ring gear was 2.5. The load-life factor of all five gears was 8.93. The planet bearings were 300 series, single-row ball bearings, with a Weibull slope of 1.1, a load-life factor of 3.0 and a life adjustment factor of 6.

Planetary Design Inequality Constraints
Table 1

Constraint	Value	Unit	Type
bending stress:sun-planet	40,000.000	psi	upper
full load Hertz stress:sun-planet	180,000.000	psi	upper
gear tip Hertz pressure:			

Design Service Lives
Table 2

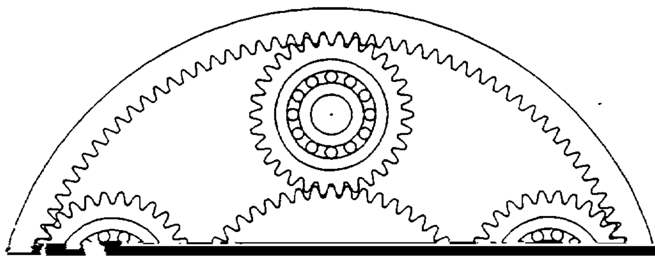
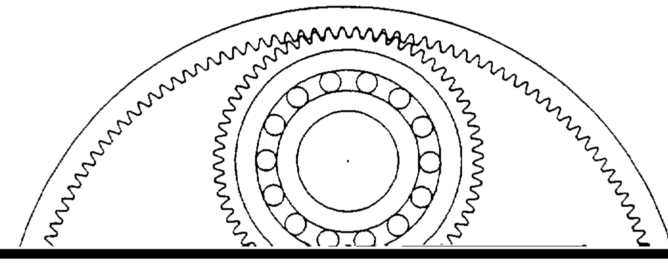
planet number	ratio	tooth numbers			face width	pitch	life	pitch	life
	n_a	N_s	N_{pl}	N_r	f	P_d	ℓ_{av}	P_d'	ℓ_{av}'
					in	in^{-1}	hrs	in^{-1}	hrs
three	3.0	60	30	120	1.0	11	1040	10.68	1320
	3.5	48	36	120	1.0	11	2430	10.7	3000
	4.0	45	45	135	1.25	12	4720	11.95	4870
	4.5	40	50	140	1.5	13	3880	12.4	5500
	5.0	36	54	144	1.5	13	4940	12.7	5870
	5.5	36	63	162	1.5	15	4300	14.2	6230
	6.0	30	60	150	1.5	14	3600	13.2	5590
	6.5	24	54	132	1.5	12	3740	11.68	4560
	7.0	24	60	144	1.5	13	3870	12.7	4600
	7.5	24	66	156	1.5	14	3900	13.7	4580
	8.0	24	72	168	1.5	15	3810	14.7	4450
four	3.0	60	30	120	1.0	11	1850	10.68	2340
	3.5	40	30	100	1.0	10	1640	9.02	3680
	4.0	40	40	120	1.25	11	5880	10.7	7240
	4.5	40	50	140	1.5	13	6900	12.35	10100
	5.0	36	54	144	1.5	13	8780	12.68	10560
five	3.0	60	30	120	1.0	11	2890	10.68	3660
	3.5	40	30	100	1.0	10	2570	9.05	5600
	4.0	40	40	120	1.25	11	9180	11.7	11300

Table 2 lists the obtained designs with the numbers of teeth on the sun, planet and ring gears, the gear face width and the diametral pitch for each ratio. These teeth numbers are discrete values which produce the required planetary ratio and allow symmetric placement of the planets for radial load cancellation. After the diametral pitch, the mean service life of the transmission is listed for component replacement at repair. This life corresponds to the integer diametral pitch listed before it. It also corresponds to a somewhat smaller transmission as dictated by the integer pitch. The last two columns show larger lives which vary more continuously and the fractional diametral pitch required to obtain these lives by allowing the transmission to have the full 12 inch outside ring diameter. The table includes blocks of data for three, four and five planet designs.

Even higher lives would be possible with fine pitch gearing since

The effect of the gear ratio on the mean life of the transmission is plotted in Figure 5. For the integer diametral pitch designs with three planets, the mean service life, plotted as a series of crosses, increased from 1040 hours for a gear ratio of three to 4940 hours for a gear ratio of five, and then decreases to 3600 hours for a gear ratio of six, with a final life of 3810 hours for a gear ratio of eight. Higher lives which varied more continuously were available with uneven pitches and are plotted as a life limit line above the found design lives. This line corresponds to the primed pitches and lives of table 2 and is also jagged due to the discrete nature of the numbers of teeth.

Similar data is plotted with circles for integer pitch designs with four planets and with squares for designs with five planets. For the four planet designs, the integer pitch design lives ranged from 1850 hours for a gear ratio of three to a maximum of 8780 hours for a gear ratio of five.

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M. Savage, K.L Rubadeux, and H.H. Coe

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NASA Lewis Research Center
Cleveland, Ohio 44135-3191
and
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